The neutral top-pion and the lepton flavor violating Z decays $Z \to l_i l_j$

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Abstract

Taking into account the constraints of the present experimental limit of the process $\mu \to e\gamma$ on the free parameters of topcolor-assisted technicolor(TC2) models, we calculate the contributions of the neutral top-pion to the lepton flavor violating(LFV) Z decays $Z \to l_i l_j$. Our results show that the value of the branching ratio $Br(Z \to \tau l)$ is larger than that of $Br(Z \to \mu e)$ in all of the parameter space and there is $Br(Z \to \tau\mu) \approx Br(Z \to \tau e) \simeq 2 \times 10^{-14}$, which is far from the reach of present or future experiments.

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It is well known that the individual lepton numbers L_e, L_μ and L_τ are automatically conserved and the tree-level lepton flavor violating(LFV) processes are absent in the standard model(SM). However, the solar neutrino experiments[1] and the atmospheric neutrino experiments[2] confirmed by reactor and accelerator experiments[3] provide very strong evidence for mixing and oscillation of the flavor neutrinos, which presently provide the only direct observation of physics that can not be accommodated within the SM and imply that the separated lepton numbers are not conserved. Thus, the SM requires some modification to account for the pattern of neutrino mixing, in which the LFV processes are allowed. The observation of these LFV processes would be a clear signature of new physics beyond the SM at the present or future experiments. This fact has made one to be of interest in the LFV processes. Among them, the LFV Z decays, such as $Z \to l_i l_j (l_i = e, \mu$ or $\tau)$, are interest subjects. Furthermore, the Giga Z option of the TESLA linear collider project will work at the resonance and increase the production rate of Z boson[4]. This forces one to study the LFV Z decays precisely.

To completely avoid the problems arising from the elementary Higgs field in the SM, various kinds of dynamical electroweak symmetry breaking(EWSB) models have been proposed, and among which the topcolor scenario is attractive because it explains the large top quark mass and provides possible dynamics of EWSB. Topcolor-assisted technicolor(TC2) models[5], flavor-universal TC2 models[6], top see-saw models[7], and top flavor see-saw models[8] are four of such examples. The presence of the physical top-pions in the low-energy spectrum is an inevitable feature of these kinds of models[9]. Studying the possible signatures of the top-pions at present and future high- or low-energy colliders can help the collider experiments to search for top-pions, test topcolor scenario and further to probe EWSB mechanism.

The branching ratios of the LFV processes $Z \to l_i l_j$ are extremely small even in the SM with massive neutrinos: $Br(Z \to l_i l_j) \leq 10^{-54}$[10], which are far from the reach of present or future experiments. The current experimental limits obtained at LEPI are[11]:

$$Br(Z \to \mu e) \quad < \quad 1.7 \times 10^{-6},$$

$$
\begin{aligned}
Br(Z \to \tau e) &< 9.8 \times 10^{-6}, \\
Br(Z \to \tau\mu) &< 1.2 \times 10^{-5},
\end{aligned}
\tag{1}
$$

and with the improved sensitivity at TESLA, these limits could be pulled down to [12]:

$$
\begin{aligned}
Br(Z \to \mu e) &< 2 \times 10^{-9}, \\
Br(Z \to \tau e) &< f \times 1.5 \times 10^{-8}, \\
Br(Z \to \tau\mu) &< f \times 2.2 \times 10^{-8},
\end{aligned}
\tag{2}
$$

with $f = 0.2 \sim 1.0$.

Recently, there are many studies on the LFV Z decays $Z \to l_i l_j$ in various models[13]. For example, these processes are investigated in a model independent way[14], supersymmetric models[15], the general two Higgs doublet model[16], the Zee model[17], and theories with a heavy boson Z'[18]. In Ref.[19], we study the contributions of the non-universal gauge boson Z' to the LFV Z decays $Z \to l_i l_j$. We find that the branching ratios $Br(Z \to \tau e)$ and $Br(Z \to \mu e)$ can approach the experimental upper limits in a sizable of the parameter space of the flavor-universal TC2 models. The aim of this paper is to consider the contributions of the neutral top-pion π_t^0 to the LFV Z decays $Z \to l_i l_j$ in the context of TC2 models, and see whether π_t^0 can give significant contributions on these processes.

For TC2 models[5], TC interactions play a main role in breaking the electroweak symmetry. Topcolor interactions make small contributions to EWSB and give rise to the main part of the top quark mass, $(1-\varepsilon)m_t$, with the parameter $\varepsilon << 1$. Thus, there is the following relation:

$$
\nu_\pi^2 + F_t^2 = \nu_W^2, \tag{3}
$$

where ν_π represents the contributions of TC interactions to EWSB, $\nu_W = \nu/\sqrt{2} \approx 174GeV$, and $F_t = 50GeV$ is the physical top-pion decay constant. This means that the masses of the gauge bosons W and Z are given by absorbing the linear combination the top-pions and technipions. The orthogonal combination of the top-pions and technipions remains unabsorbed and physical. However, the absorbed Goldstone linear

combination is mostly the technipions while the orthogonal combination is mostly the top-pions, which are usually called physical top-pions($\pi_t^{\pm}, \pi_t^0$). The flavor diagonal(FD) couplings of the neutral top-pion π_t^0 to leptons can be written as[9,20]:

$$\frac{m_l}{\nu}\bar{l}\gamma^5 l\pi_t^0, \tag{4}$$

where $l = \tau, \mu$, and e.

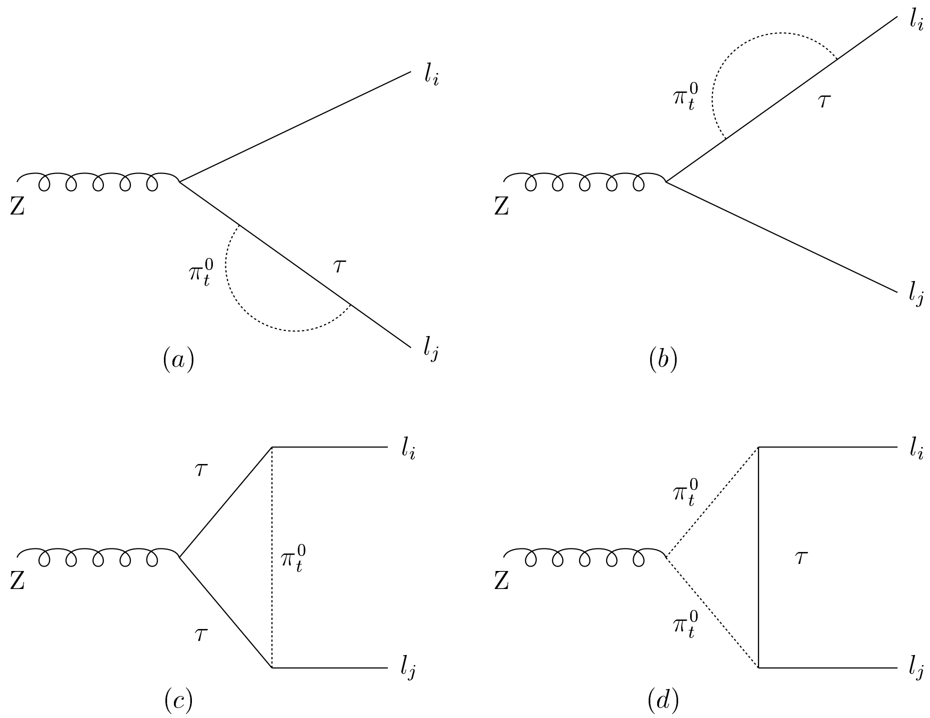

Figure 1: Feynman diagrams contribute to the LFV processes $Z \to l_i l_j$ due to the neutral top-pion π_t^0 exchange.

For TC2 models, the underlying interactions, topcolor interactions, are non-universal and therefore do not posses GIM mechanism. The non-universal gauge interactions result in the flavor changing(FC) coupling vertices when one writes the interactions in the mass eigenbasis. Thus, the top-pions can induce the new FC scalar coupling vertices[21]. The

FC couplings of π_t^0 to leptons can be written as:

$$\frac{m_\tau}{\nu} k_{\tau i} \overline{\tau} \gamma^5 l_i \pi_t^0, \tag{5}$$

where $l_i (i = 1, 2)$ is the first(second) lepton $e(\mu)$, $k_{\tau i}$ is the flavor mixing factor, which is the free parameter. Certainly, there is also the FC scalar coupling $\pi_t^0 \mu e$. However, the topcolor interactions only contact with the third generation fermions. The flavor mixing between the first and second generation fermions is very small, which can be ignored.

From above discussions, we can see that the π_t^0 can indeed induce the LFV Z decays $Z \to l_i l_j$ via the FC scalar couplings $\pi_t^0 \tau l_i$. The relevant Feynman diagrams are depicted in Fig.1. The internal fermion line may be the leptons τ, μ or e. However, the internal fermion propagator provides a term proportional to m_l^2 in the numerator, which is not cancelled by the m_l^2 in the denominator since the heavy π_t^0 mass m_{π_t} dominates the denominator. Thus, we only take the internal fermion line as the τ lepton line.

Using Eq.(4), Eq.(5) and other relevant Feynman rules, the effective vertex $Z\overline{\mu}e$ contributed by π_t^0 exchange can be written as:

$$\Lambda_{Z\overline{\mu}e} = \frac{iek_{\tau\mu}k_{\tau e}m_\tau^2}{16\pi^2\nu^2}[\gamma^\mu(F_1 + F_2\gamma^5) + P_\mu^\mu(F_3 + F_4\gamma^5) + P_e^\mu(F_5 + F_6\gamma^5)], \tag{6}$$

where $S_W = \sin\theta_W$, θ_W is the Weinberg angle. The form factor F_i can be written as:

$$\begin{aligned}
F_1 &= \frac{4S_W^2 - 1}{4S_W C_W}[-\frac{m_e}{m_\mu}B_1' - \frac{m_\tau}{m_\mu}B_0' - B_1'' + \frac{m_\tau - m_\mu}{m_\mu}B_0'' - B_0''' - m_{\pi_t}^2 C_0' \\
&\quad -m_\tau(m_\tau - m_\mu)C_0' + 2C_{24}' - m_e(m_\tau - m_\mu)(C_{11}' - C_{12}') + m_\tau m_\mu C_{12}' \\
&\quad -m_\tau m_e(C_{11}' - C_{12}') + m_\mu(m_\tau - m_\mu)C_{12}'], \\
F_2 &= \frac{1}{4S_W C_W}[-\frac{m_e}{m_\mu}B_1' - \frac{m_\tau}{m_\mu}B_0' - B_1'' + \frac{m_\tau - m_\mu}{m_\mu}B_0'' + B_0''' + m_{\pi_t}^2 C_0' \\
&\quad -m_\tau(m_\tau - m_\mu)C_0' - 2C_{24}' - m_e(m_\tau - m_\mu)(C_{11}' - C_{12}') + m_\tau m_\mu C_{12}' \\
&\quad +m_\tau m_e(C_{11}' - C_{12}') - m_\mu(m_\tau - m_{l_i})C_{12}' - 4C_{24}''], \\
F_3 &= \frac{4S_W^2 - 1}{4S_W C_W}[-2m_e(C_{22}' - C_{23}') + 2m_\mu C_{22}' - 2(m_\tau - m_\mu)C_{12}'], \\
F_4 &= \frac{1}{4S_W C_W}[-2m_e(C_{22}' - C_{23}') - 2m_\mu C_{22}' + 2(m_\tau - m_\mu)C_{12}' + 2m_\tau C_0'' \\
&\quad -2m_e(C_{11}'' - C_{12}'') + 2m_\mu C_{12}'' + 4m_\tau C_{12}'' + 4m_e(C_{22}'' - C_{23}'') + 4m_\mu C_{22}''],
\end{aligned} \tag{7}$$

$$
\begin{aligned}
F_5 &= \frac{4S_W^2-1}{4S_WC_W}[-2m_e(C'_{21}+C'_{22}-2C'_{23})+2m_\mu(C'_{22}-C'_{23})-2m_\tau(C'_{11}-C'_{12})], \\
F_6 &= \frac{1}{4S_WC_W}[-2m_e(C'_{21}+C'_{22}-2C'_{23})-2m_\mu(C'_{22}-C'_{23})-2m_\tau(C'_{11}-C'_{12}) \\
&\quad -2m_\tau C''_0+2m_e(C''_{11}-C''_{12})-2m_\mu C''_{12}-4m_\tau(C''_{11}-C''_{12}) \\
&\quad +4m_e(C''_{21}+C''_{22}-2C''_{23})+4m_\mu(C''_{22}-C''_{23})].
\end{aligned}
$$

The two- and three-point Feynman integrals B_n, C_0 and C_{ij} can be written as[22]:

$$
\begin{aligned}
B'_n &= B_n[-P_e, m_{\pi_t}, m_\tau], \qquad B''_n = B_n[-P_\mu, m_{\pi_t}, m_\tau], \\
B'''_n &= B_n[-P_Z, m_\tau, m_\tau], \qquad C'_0 = C_0[P_e, -P_Z, m_{\pi_t}, m_\tau, m_\tau], \\
C''_0 &= C_0[P_e, -P_Z, m_\tau, m_{\pi_t}, m_{\pi_t}], \\
C'_{ij} &= C_{ij}[P_e, -P_Z, m_{\pi_t}, m_\tau, m_\tau], \\
C''_{ij} &= C_{ij}[P_e, -P_Z, m_\tau, m_{\pi_t}, m_{\pi_t}].
\end{aligned} \tag{8}
$$

Where p_μ and p_e denote the momenta of the two final state leptons μ and e, p_Z denotes the momentum of the gauge boson Z. For the case of calculating the contributions of π_t^0 to the LFV Z decays $Z \to l_i l_j$, the process $Z \to \tau l$ (l=μ or e) is similarly to the process $Z \to \mu e$. The differences between $Z \to \mu e$ and $Z \to \tau l$ are the final state particles and the FC scalar coupling forms. The neutral top-pion π_t^0 generates the LFV process $Z \to \mu e$ via the FC couplings $\pi_t^0 \tau\mu$ and $\pi_t^0 \tau e$, but the LFV process $Z \to \tau e$ $(\tau\mu)$ is induced by the FC coupling $\pi_t^0 \tau e$ $(\pi_t^0 \tau\mu)$. Thus, the effective vertices $\Lambda_{Z\overline{\tau}e}$ and $\Lambda_{Z\overline{\tau}\mu}$ can be written as:

$$
\Lambda_{Z\overline{\tau}e} = \Lambda_{Z\overline{\mu}e}(m_\mu \to m_\tau, k_{\tau\mu}k_{\tau e} \to k_{\tau e}), \tag{9}
$$

$$
\Lambda_{Z\overline{\tau}\mu} = \Lambda_{Z\overline{\tau}e}(m_e \to m_\mu, k_{\tau e} \to k_{\tau\mu}). \tag{10}
$$

In above equations, we have taken into account all the masses of internal lepton τ and external leptons(anti-leptons).

In general, the decay width of the LFV process $Z \to l_i l_j$ can be written as:

$$
\Gamma = \int \frac{(2\pi)^4}{6m_Z}\delta^4(P_Z - q_1 - q_2)\frac{d^3q_1}{(2\pi)^3 2E_1}\frac{d^3q_2}{(2\pi)^3 2E_2}|M|^2(P_Z, q_1, q_2), \tag{11}
$$

where M is the amplitude of the process $Z \to l_i l_j$. q_1 and q_2 denote the momenta of the two final state leptons.

To obtain numerical results, we take the SM parameters as $\alpha(m_Z) = \frac{1}{128.8}$, $S_W^2 = 0.2315$, $m_\tau = 1.777GeV$, $m_\mu = 0.105GeV$ and m_e=0 [11]. The limits on the mass m_{π_t} of the top-pion may be obtained via studying its effects on the various experimental observables[9]. It has been shown that m_{π_t} is allowed to be in the range of a few hundred GeV depending on the models. As numerical estimation, we take the top-pion mass m_{π_t} as a free parameter.

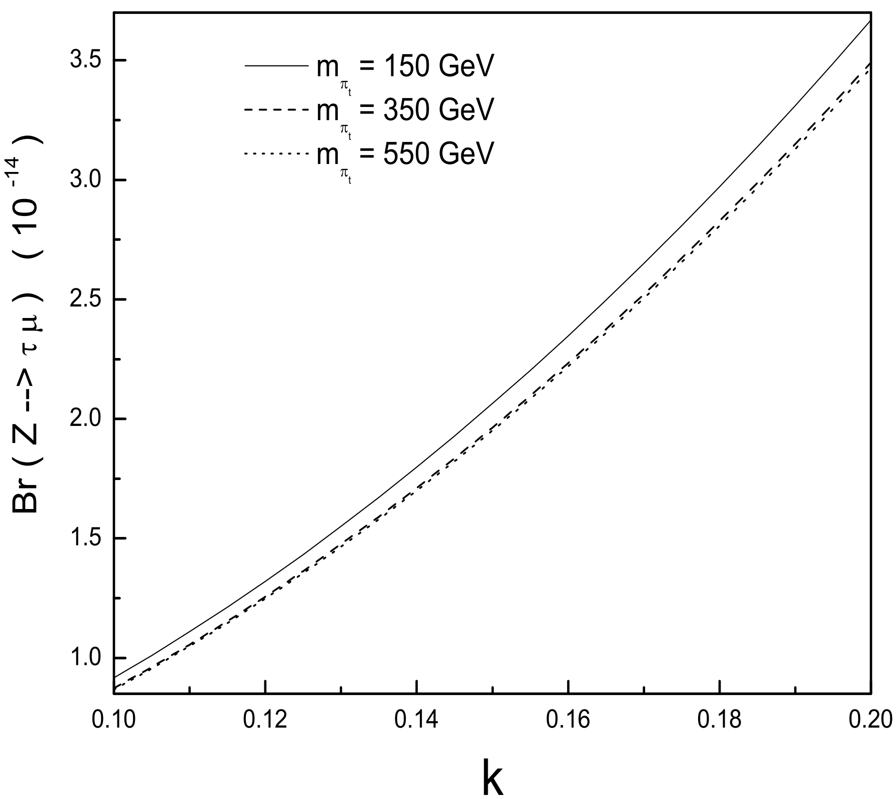

Figure 2: The branching ratios of the LFV process $Z \to \tau\mu$ as a function of the flavor mixing factor k for three values of the top-pion mass m_{π_t}.

For TC2 models, the topcolor interactions only contact with the third generation. The new particles, such as extra gauge boson Z' and top-pions $\pi_t^{0,\pm}$, treat the fermions in the third generation differently from those in the first and second generation and treat the fermions in the first generation same as those in the second generation. So, we can assume

that the flavor mixing factor $k_{\tau\mu}$ is equal to the flavor mixing factor $k_{\tau e}$: $k = k_{\tau\mu} = k_{\tau e}$. In this case, we have $Br(Z \to \tau\mu) \approx Br(Z \to \tau e)$ for $m_\mu \approx 0$ and $m_e \approx 0$.

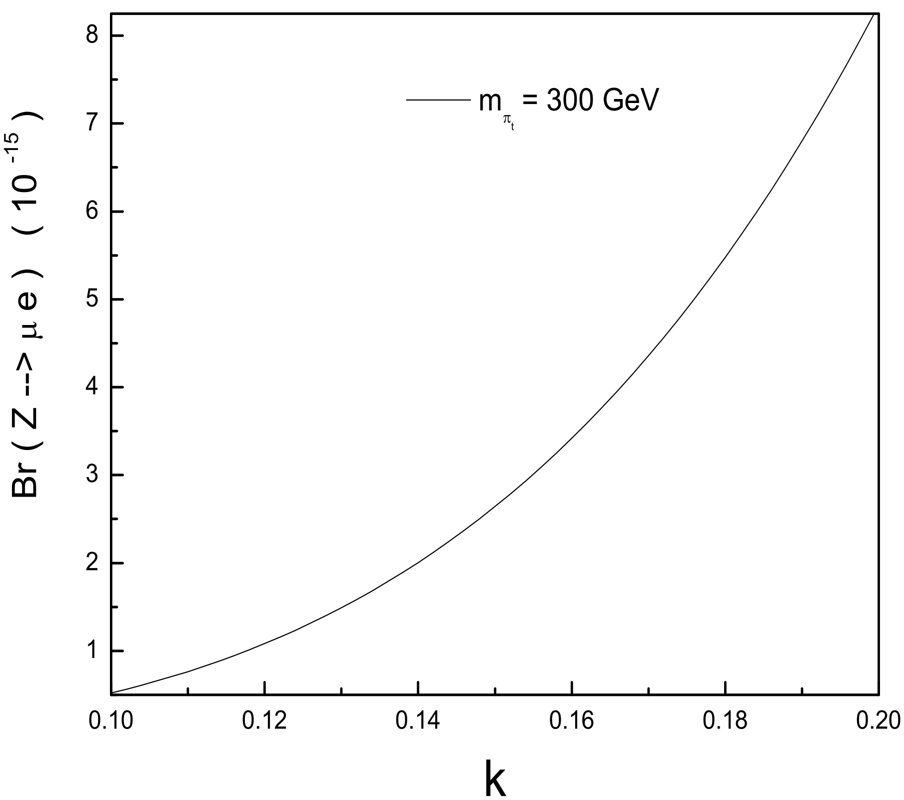

Figure 3: The branching ratio $Br(Z \to \mu e)$ as a function of the flavor mixing factor k for $m_{\pi_t} = 300GeV$.

The neutral top-pion π_t^0 can give significant contributions to the LFV proscess $l_i \to l_j\gamma$ and $l_i \to l_j l_k l_l$[23]. The present experimental bound on $\mu \to e\gamma$ gives severe constraint on the mixing factor k. If we assume that the π_t^0 mass m_{π_t} is smaller than $400GeV$, then there must be $k \leq 0.2$[23]. Taking into account the constraint on the parameter k, the branching ratio $Br(Z \to \tau l)(l = \mu$ or $e)$ is plotted in Fig.2 as a function of the parameter k for three values of the top-pion mass m_{π_t}. From Fig.2 one can see that the value of the branching $Br(Z \to \tau l)$ increases as k increasing and is insensitive to m_{π_t}.

For $200GeV \leq m_{\pi_t} \leq 400GeV$ and $0.1 \leq k \leq 0.2$, the branching ratio $Br(Z \to \tau l)$ is in the range of $8.7 \times 10^{-15} \sim 3.6 \times 10^{-14}$. The branching ratio $Br(Z \to \mu e)$ is shown in Fig.3 as a function of k for $m_{\pi_t} = 300GeV$. For $k = 0.2$, the value of the branching ratio $Br(Z \to \mu e)$ can reach 8.3×10^{-15}.

In general, topcolor scenario predicts the existence of the non-universal gauge boson Z' and the top-pions $\pi_t^{\pm,0}$. These new particles can induce the FC couplings, which have significant contributions to the LFV processes. In Ref.[19], we have studied the contributions of the non-universal gauge boson Z' to the LFV Z decays $Z \to l_i l_j$ in the context of the flavor-universal TC2 models and TC2 models. Considering the constraint of the $B\overline{B}$ mixing on the Z' mass $M_{Z'}$, we find that, in most of the parameter space of TC2 models, there are $Br(Z \to \tau\mu) \approx Br(Z \to \tau e) < 1 \times 10^{-11}$ and $Br(Z \to \mu e) < 1 \times 10^{-13}$. Thus, the contributions of the TC2 models to the LFV Z decays $Z \to l_i l_j$ mainly come from the non-universal gauge boson Z'. This is because the couplings of the neutral top-pion π_t^0 to leptons are very small. They are proportional to the factor $\frac{m_l}{\nu}$, in which m_l is the lepton mass and $\nu \approx 246\ GeV$.

High energy e^+e^- colliders can be used as Z factory, providing an opportunity to examine the decay properties of the gauge boson Z in detail. The improved experimental measurements at present stimulate the studies of the Z decays. For example, with the Giga-Z option of the TESLA linear collider project, one may expect the production of about $10^9 Z$ bosons at resonance[4]. The huge rate allows one to study a number of problems with unprecedented precision. Among them is the search for the LFV Z decays $Z \to l_i l_j$. Ref.[23] has shown that the present experimental bound on the LFV process $\mu \to e\gamma$ produce severe constraints on the free parameters of TC2 models. In this paper, based on these constraints, we calculate the contributions of the neutral top-pion π_t^0 to the LFV Z decays $Z \to l_i l_j$ via the FC scalar couplings $\pi_t^0 \tau l_i$. We find that the branching ratio $Br(Z \to \mu e)$ is smaller than the branching ratio $Br(Z \to \tau\mu)$. In wide range of parameter space of TC2 models, there is $Br(Z \to \tau e) \approx Br(Z \to \tau\mu) \sim 10^{-14}$, which is far from the reach of present or future experiments.

Acknowledgments

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